AP Physics C: Mechanics
Mr. Fletcher

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AP Review Packet Lab

Chapter 8: Potential Energy

Background Information / Chapter Summary:

Chapter 8 of an AP Physics Mechanics textbook focuses on the concept of potential energy, a fundamental quantity in classical mechanics. Potential energy plays a critical role in understanding the dynamics of objects and systems in various physical scenarios.

Important Major Topics:

Potential Energy is the energy associated with the position or configuration of objects in a conservative force field. It represents the ability of a system to do work based on its position relative to some reference point. Potential energy is a scalar quantity and is measured in joules (J).

Gravitational Potential Energy is one of the most common forms of potential energy encountered in mechanics is gravitational potential energy. It arises from the gravitational force acting on an object near the surface of the Earth. The gravitational potential energy U_g of an object of mass m at a height h above a reference point is given by: $U_g = mgh$

where g is the acceleration due to gravity (approximately 9.81 m/s^2)

Elastic Potential Energy is another important form of potential energy, which arises in systems involving springs or elastic materials. The elastic potential energy *U* stored in a spring that has been stretched or compressed a distance *x* from its equilibrium position is given by: $Ue = \frac{1}{2}kx^2$ where *k* is the spring constant, a measure of the stiffness of the spring.

Conservation of Mechanical Energy is one of the key concepts related to potential energy. In a conservative force field, the total mechanical energy (the sum of kinetic and potential energy) of a system remains constant over time, provided there are no non-conservative forces (such as friction) acting on the system.

Common Equations:

Change in Potential Energy:
$\Delta U_g = mg\Delta h$

Potential Energy w/ a Spring:
$U_S = \frac{1}{2}k(\Delta x)^2$

Potential Difference:
$\Delta U = U_f - U_i$

Conservation of Energy
$K_1 + U_1 = K_2 + U_2$

Potential Energy (Gravity):
$U_G = -(Gm_1m_2)/(r)$

Important Notes / Additional Information:

Derivations of Potential Energy Equations:

Calculating *U*

For gravity:

$$W_g = \int_{x_i}^{x_f} F_g \bullet \, dx$$

$$W_g = \int_{y_i}^{y_f} -mg \bullet \, dy \quad (\text{where } y_i > y_f)$$

$$W_g = mgy_i - mgy_f$$

$$\boxed{U_g \equiv mgh}$$

Calculating *U*

For a spring:

$$W_{spring} = \int_{x_i}^{x_f} F \bullet dx$$

$$W_{spring} = \int_{x_i}^{x_f} -kx \bullet dx$$

$$W_{spring} = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

$$W_{spring} = U_i - U_f \text{ where } \boxed{U_s \equiv \frac{1}{2}kx^2}$$

Practice Problems:

1. The behavior of a non-linear spring is described by the relationship $F = -2kx^3$, where x is the displacement from the equlibrium position and F is the force exerted by the spring. How much potential energy is stored in the spring when it is displaced a distance x from equilibrium?

Solution:

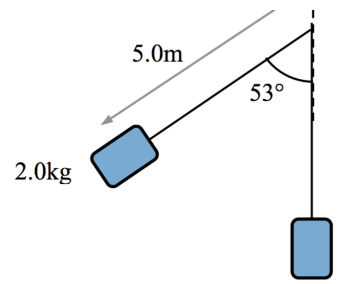
The potential energy stored in the spring is calculated using the Work integral:

$$U = -\int_{x_i}^{x_f} F \bullet dx$$

$$U = -\int_{0}^{x} -2kx^3 \bullet dx$$

$$U = 2k \left. \frac{x^4}{4} \right|_0^x = \frac{1}{2}kx^4$$

2.



A mass of 2.0 kg is attached to the end of a light cord to make a pendulum 5.0 meters in length. The mass is raised to an angle of 53° relative to the vertical, as shown, and released. The speed of the mass at the bottom of its swing is:

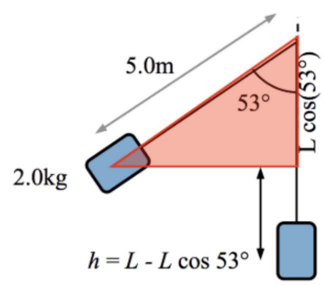
Solution:

This is a conservation of energy problem, with the gravitational potential energy *U* of the pendulum bob converted to kinetic energy *K* as it swings down. Let's consider the lowest position of the pendulum to be $h = 0$:

$$U_i + K_i = U_f + K_f$$

$$mgh + 0 = 0 + \frac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$



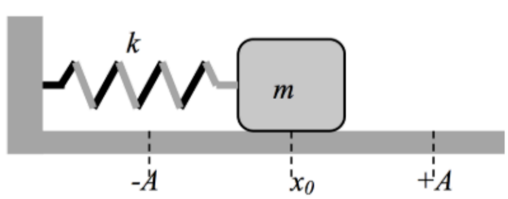
We can find the height of the pendulum bob relative to the bottom of its swing by using trigonometry. The length of the short side of the triangle (shown in red) is $L\cos 53°$. The height h is the full length L less this leg of the triangle.

$$h = L - L\cos\theta = 5 - 5\cos 53° = 2m$$

We can then use this information in our original formula to determine the velocity at this point:

$$v = \sqrt{2gh} = \sqrt{2(10m/s^2)(2m)} = 6.3m/s$$

3.



A spring with negligible mass and spring constant *k* is attached on one end to a block of mass *m*, and fastened at the other end to a wall. The block is pulled back a distance *A* from its equilibrium position and released so that it oscillates on the frictionless, horizontal surface. What is the velocity *v* of the mass as it passes the equilibrium position x_0?

Solution:

This is a conservation of energy problem, with the mass-spring's elastic potential energy at the endpoints, $U_{spring} = \frac{1}{2}kx^2$, converting completely to kinetic energy at the midpoint, $K = \frac{1}{2}mv^2$.

$$U_s = K_0$$

$$\frac{1}{2}kA^2 = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{kA^2}{m}} = A\sqrt{\frac{k}{m}}$$